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ABSTRACT

This is a project in an emerging line of research investigating children's informed knowledge of mathematics questions. The purpose of this study was to analyze the ability of students who had not received multiplication or division instruction to solve multiplication and division word problems. The study consisted of videotaped interviews with 89 first, second, and third grade students. The students read two multiplication problems, two partitive division problems, and two measurement division problems. They were asked to solve the problems in two different environments; the first three problems mentally, and the final three with paper and pencil. Significant differences were found in performance level among the grades and across the problem types. No significant differences were found between the two environments. Included are 3 tables and 13 references. Appendices include questions, a coding sheet, and a consent form. (DC)

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# Achievement of First-, Second-, and Third-Grade Students on Multiplication and Division Word Problems in Two Different Solution Environments

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June 1, 1989

This study was done under the direction of Vicky Kouba and Roger Boothroyd as partial completion of a Master's Degree in Mathematics Education at the State University of New York of Albany.

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ABSTRACT

The purpose of this study was to analyze the ability of students who had not received multiplication or division instruction or who had just begun multiplication instruction to solve multiplication and division word problems. The study consisted of video-taped interviews with eighty-nine 1st-, 2nd-, and 3rd-grade students. The students were read two multiplication problems, two partitive division problems and two measurement division problems. The students were asked to solve the problems in two different environments: they were asked to solve the first three problems mentally and were provided with paper and pencil for the final three problems. Significant differences were found in performance level among the grades and across the problem types. No significant difference was found between the two environments.

# INTRODUCTION

This project is one in an emerging line of research investigating children's informal knowledge of mathematics operations. Information gathered through this research can be used to develop a better understanding of how children gain an understanding of operations and numbers and can be used to design curriculum and in-service programs to help teachers build on children's informal knowledge (Hiebert & Behr, 1988).

At a time when the American educational system has come under heavy attack, research such as this demonstrates that children can do more than we expect of them and are, in fact, good problem solvers (Moser & Carpenter, 1982).

This particular research project is an extension of previous research and methodology in the area of multiplication and division (Kouba, 1987).

Variable Definitions

Independent variables. In this study, three independent variables were controlled. The first was grade level (1st grade, 2nd grade, and 3rd grade). Second, students were presented three types of problems: multiplication, measurement division, and partitive division. A multiplication problem was one where the product was unknown. A measurement division problem was one where the number of sets, or groups, was unknown. A partitive division problem was one where the number of elements in each set or group was unknown (see Appendix 1 for specific problems). The classification and format of these problems was based on research completed by Dr. Kouba (Kouba, 1987), thereby insuring construct validity in our study. Finally, students were asked to solve these problems in two different environments: mentally and with paper and pencil. Within the mental environment, students were provided no physical objects to manipulate or paper and pencil to use. This did not preclude the use of fingers and students were not discouraged from using their fingers. In the

paper and pencil environment students were encouraged to use the paper and pencil in any way that might help them think about the problem.

Dependent variable. The dependent variable was the level of success of the sample, rather than the individual. This was measured in two ways: by the percentage of students who were able to respond with a correct answer and by the number of students who either responded with a correct answer or demonstrated the use of a correct strategy that would have lead them to correct answers except that they made some minor error.

## Research Questions

The purpose of this research was to explore the following questions: To what extent are 1st, 2nd, and 3rd grade students able to solve multiplication and division word problems? How do the type of problem and the solution environment affect this performance?

## Literature Review

Very little research has been done on the ability of first and second grade students to solve multiplication and division word problems.

The subjects in most multiplication and division studies have been students between the ages of 10 and 15 (DeCorte, Lieven, & VanCollie, 1988). Third grade students have been included in the Mathematics Assessment of the National Assessment of Educational Progress. However, in the first three assessments, they were not administered any division items. In these assessments, 25% to 66% of third grade students were able to successfully complete written problems involving multiplication of a two digit number by a one digit number (Carpenter, Coburn, Reys & Wilson, 1978; Carpenter, Corbitt, Kepner, Linquist & Reys, 1981; National Assessment of Educational Progress, 1984; Brown, Carpenter, Kouba, Linquist, Silver, & Swafford, in press). In the fourth Assessment, 21% of the third grade students were able to successfully complete written problems involving division of a two-digit number by a one-digit number (Brown, et al., in press).

Many researchers have reported that children find division tasks more difficult than multiplication tasks (Suydam & Weaver, 1981; Close & Murtagh, 1986). In addition, partitive division

tasks are more difficult for children than measurement division tasks (Gunderson, 1953; Zweng, 1964; Vest, 1986).

Children are provided with manipulative or paper and pencil in most of the studies cited here, but we could find no research on the effects that different environments have on students' ability to solve multiplication and division word problems.

## METHODS

### Subjects

The sample for this study consisted of first, second, and third grade students from a small, northeastern elementary school. All students received mathematics instruction from their classroom teachers. Parents and students were asked to sign a form granting permission to participate in the study. Since the interviews were video taped and also because there is a possibility that the video tapes may be used in research presentations or other appropriate educational settings, the consent form also

included a "model release" (see Appendix 3 for consent form).

Of the 159 consent forms that were sent home, 141 were returned. 89 students consented and were granted permission by a parent to participate in the study. Classroom teachers were asked to classify non-participants according to their overall mathematics achievement. From their reports it appears that the non-participants were scattered across all levels of mathematics achievement. Thus, the non-participant population did not differ from the participant population in terms of mathematics achievement.

Organization of Tasks

One hundred sets of problems were constructed for the interviews. Each set consisted of six questions, two each of multiplication, partitive division, and measurement division.

To avoid any confounding variables, each type of problem was randomly ordered within each set, keeping one problem of each type within each half of the set. This was done by assigning each problem a number and then using a random number table. Secondly the number triples used in each

problem were randomly assigned. The six number triples used were:

3,6,18  3,7,21  3,8,24  3,9,27  4,6,24  4,7,28

To avoid assigning three triples containing the number 3 to one half of the set, we used the same type of grouping as we had when assigning a problem type to each half of the set. Therefore, each half of the set contained a number triple with the number 4. Finally, the number which was assigned to the objects in each problem was randomly chosen as either the larger or the smaller of the factors. This randomization was limited so that one half of the set of six problems would not contain all large numbers of objects or all small numbers of objects. After the first three problems were assigned, the same problem types in the second half of the problem set were assigned the opposite large/small number of objects designation. The next problem set was the complement of this set in its large/small configuration.

Procedure

In February, we conducted a pilot study with 13 first, second, and third grade students, thus assuring content validity for our study. As a result of that study, the wording of the partitive division problems was changed so that the question flowed more smoothly. From the video tapes of these interviews we piloted our coding form and established interrater reliability of .876. In establishing this reliability coefficient we compared only our coding of the error responses, which we felt was the only subjective component of our coding sheet. Had we included other sections of the coding sheet (e.g. whether the student's answer was correct or if the student amended his or her answer) we would have most likely obtained an even higher reliability coefficient.

In March, consent forms were sent home with all students in first, second, or third grade. During the following week we spoke with each of the classes. We told the students that we wanted to find out how they think when they do a mathematics problem and that we would ask them a few questions involving numbers. After they

answered the question, we would ask them what kinds of things they were thinking about while they were solving the problem, and that we would be video taping the interviews to help us remember their responses. The students were also told that we would like their help with our "homework," but that they could change their minds about participating at any time. They were assured that the interview had nothing to do with the school or their grades and that the teachers would not be informed of their individual performances.

The interviews were conducted between March 13 and March 21 in a room used for reading remediation in the mornings and in the kindergarten classroom in afternoons (both of these rooms were otherwise unoccupied at the time), and in a small room adjacent to one of the first grade classrooms. To avoid any peer pressure from classmates, only second and third grade students were interviewed in this room.

The interviewer walked with each student from the classroom to the interviewing site. During this time the interviewer asked the student questions like, "Do you have any pets?", "Do you

have brothers or sisters at this school?", or "When is your birthday?" in order to establish a good relationship with the student. After arriving at the interview site, the interviewer briefly reviewed the information that had been give to the students as a class during the previous week. The student was also told that the interviewer could repeat the question as many times as the student wished.

The student was then read three questions which the she/he solved mentally. After the student responded to each question, the interviewer asked questions like:

Can you explain to me how you got that answer?
What numbers did you think of?
What number did you think of first?
How did you know when to stop?
It looked as if you were doing something with your fingers. Can you show me what you did?
Was there something in the problem that told you to do what you did?

If the student said something like, "I counted it out," the interviewer responded, "Can you count it out loud for me?"

Before beginning the final three questions the student was provided with paper and pencil and told that it was "scrap" paper and that the student could use it in any way that might be

helpful to her/him. After the students responded to each question, the interviewer asked questions similar to those listed above as well as:

Can you tell me about what you wrote?
Is there a difference between having paper and pencil and having to solve the problems without them?

Throughout the interview, the interviewer made notes on a coding form for each question. Notes were made on what the student said, did, or wrote, as well as noting whether the student's answer was correct, whether or not the student amended her/his answer, as well as the type of any error that was made (see Appendix 2 for coding form and explanation of codes).

At the conclusion of the interview, the interviewer thanked the child, asked the child if she/he had enjoyed the interview experience, and walked the child back to her/his classroom. Of the 89 students interviewed, three students appeared excessively anxious about the interview. According to the interview protocol, if a student seemed excessively uncomfortable during the interview, the interviewer skipped to an easy problem to end the interview. The interview thus ended on a positive note for these students, but

the data from these interviews was not included in the final analysis.

Design and Analysis

Frequency counts were made of the number of students at each grade level who responded correctly to each problem type in each environment. Results are recorded in Table 1. In addition, frequency counts were made of students who responded correctly or demonstrated a correct strategy. These results are reported in Table 2. A multivariate analysis of variance was also conducted across the three variables based on the number of students who responded correctly or who demonstrated a correct strategy. These results are listed in Table 3.

## RESULTS

Significant differences in students ability to solve multiplication and division word problems were found among the grades and across the three problem types. There was no significant difference on performance across the environments.

TABLE 1

PERCENTAGE OF STUDENTS IN EACH GRADE RESPONDING WITH A CORRECT ANSWER

| GRADE / PROBLEM TYPE | 1 (N = 28) | 2 (N = 30) | 3 (N = 28) |
|---|---|---|---|
| MENTAL ENVIRONMENT | | | |
| Multiplication | 7.1 | 30.0 | 50.0 |
| Measurement Division | 10.7 | 40.0 | 39.3 |
| Partitive Division | 7.1 | 23.3 | 28.6 |
| PAPER AND PENCIL ENVIRONMENT | | | |
| Multiplication | 14.3 | 40.0 | 57.1 |
| Measurement Division | 21.4 | 33.3 | 28.6 |
| Partitive Division | 17.9 | 20.0 | 46.4 |

TABLE 2

PERCENTAGE OF STUDENTS IN EACH GRADE RESPONDING WITH CORRECT ANSWER OR DEMONSTRATING A CORRECT STRATEGY

| GRADE / PROBLEM TYPE | 1 (N = 28) | 2 (N = 30) | 3 (N = 28) |
|---|---|---|---|
| MENTAL ENVIRONMENT | | | |
| Multiplication | 21.4 | 46.7 | 75.0 |
| Measurement Division | 10.7 | 43.3 | 53.6 |
| Partitive Division | 7.1 | 36.7 | 39.3 |
| PAPER AND PENCIL ENVIRONMENT | | | |
| Multiplication | 17.9 | 40.0 | 67.9 |
| Measurement Division | 28.6 | 46.7 | 42.9 |
| Partitive Division | 21.4 | 26.7 | 53.6 |

TABLE 3

RESULTS OF MULTIVARIATE ANALYSIS OF VARIANCE OF GRADE LEVEL, PROBLEM TYPE AND SOLUTION ENVIRONMENT

| Source | DF | Sum of Squares | Mean Square | F Ratio |
|---|---|---|---|---|
| Grade Level | 2 | 11.95 | 5.97 | 9.38* |
| Environment | 1 | 0.02 | 0.02 | 0.13 |
| Problem Type | 2 | 1.69 | 0.84 | 7.33** |
| Grade x Environment | 2 | 0.46 | 0.23 | 1.42 |
| Grade x Problem Type | 4 | 1.24 | 0.31 | 2.71*** |
| Environment x Problem | 2 | 0.34 | 0.17 | 1.43 |
| Grade x Env. x Prob | 4 | 0.70 | 0.17 | 1.48 |

* $p<.001$
** $p=.001$
*** $p=.032$

## DISCUSSION

Since third grade students were receiving instruction in multiplication at the time of our study, it is not surprising that the highest percentages of students responding correctly or demonstrating a correct strategy were is the third grade multiplication cells. However, the

significant differences in performance between first and second grade students, without formal instruction in multiplication and division, should be investigated more thoroughly. These differences could be due to the fact that second grade students are starting to use the counting strategies that they were learning as first grade students.

Although there was no significant difference between the two solution environments, there appears to be an important difference in performance across the environments, particularly in the division problems for first grade students. One reason for this could be that in multiplication problems, students are given two smaller numbers in the problem and asked to provide a larger number for an answer. In division problems, students are provided with a large number and a smaller number in the problem and should respond with a small number for an answer. Many of the first grade students had difficulty keeping track of a large number mentally and compensated for this by immediately drawing some type of markers or tallies to

represent the large number when they were read the problem.

In general, students were most successful at solving multiplication problems. In solving multiplication problems, students can interpret each piece of information and then move on to the next piece of information. For example "You are making 4 strawberry shortcakes," student draws four shortcakes, "if you put 7 strawberries on each cake," student draws strawberries, "how many strawberries do you need altogether?" student counts strawberries. Except for the third grade students in the paper and pencil environment, all students demonstrated the most difficulty solving partitive division problems. In measurement division, a student needs to keep track of two pieces of information while solving the problem: the total number of objects and the number of objects that are to be placed in each group. But in partitive division problems, the student needs to remember the total number of objects, the number of groups that the objects are to be split into, and the fact that the objects are to be distributed equally.

The fact that many more students were able to demonstrate a correct strategy than were able to respond with the correct answer suggests that there is some kind of processing error being made by these students. More research into these types of processing errors could shed light onto the difficulties some students experience in solving multiplication and division problems.

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APPENDIX 1: Questions

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MULTIPLICATION

You are having soup for lunch. There are _____ bowls. If you put _____ crackers in each bowl, how many crackers do you need altogether?

You are making _____ strawberry shortcakes. If you put _____ strawberries on each cake, how many strawberries do you need altogether?

---

MEASUREMENT DIVISION

You have _____ marbles to put into bags. If you put _____ marbles into each bag, how many bags do you need altogether?

You are making hot chocolate. You have _____ marshmallows to use up. If you put _____ marshmallows into each cup, how many cups do you need?

---

PARTITIVE DIVISION

_____ friends have _____ tomatoes that they want to share equally. How many tomatoes will each person get?

_____ horses share _____ apples equally. How many apples does each horse get?

## APPENDIX 2: Coding Sheet

Problem:
You have _____ marbles to put into bags. If you put _____ marbles into each bag, how many bags do you need altogether?

---

Correct_____ Incorrect_____ Answer_____

---

Observations:
Visual

Written

Errors

| TYP | POS | TRP | L/S | #ID | SET | STU | OB# | SE# | SEX | GRA | TCH | AMD | ANS | COR | ERR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | |

*APPENDIX 2: Continued*
Explanation of Codes

TYP: Type of Problem
- M Multiplication
- MD Measurement Division
- PD Partitive Division

POS: Position in Set (1-6)

TRP: Number Triple

| | |
|---|---|
| 1 | 3, 8, 24 |
| 2 | 4, 6, 24 |
| 3 | 3, 7, 21 |
| 4 | 3, 9, 27 |
| 5 | 3, 6, 18 |
| 6 | 4, 7, 28 |

L/S: Large or Small Number of Objects
- L Large
- S Small

| | |
|---|---|
| 1 and 2+ | L, S, S |
| 2 and 1+ | S, L, L |
| 3 and 4+ | S, L, S |
| 4 and 3+ | L, S, L |
| 5 and 6+ | S, S, L |
| 6 and 5+ | L, L, S |

#ID SET STU: Set and Student Identification Number (100-190)

OB#: Number of objects in each set

SE#: Number of sets

SEX: Student's Sex (M or F)

GRA: Student's Grade (1-3)

TCH: First Letter of Teacher's Last Name

AMD: Did student amend answer? (Y or N)

COR: Was student's answer correct? (Check if correct)

ERR: Error Type
- W+ Wrong Operation: Addition
- W- Wrong Operation: Subtraction
- Wx Wrong Operation: Multiplication
- W/ Wrong Operation: Division
- I Inaccurate calculation
- F Forgets one of numbers in problem
- I/F Inaccurate calculation and forgets
- R Respond with a number from problem
- O Only identifies operation
- N Wrong number fact
- U Unidentifiable

Note: I, F and I/F were considered correct strategies.

APPENDIX 3: Consent Form

Dear Parent,

During March, interviews will be conducted with 1st, 2nd, and 3rd grade students at Walter B. Howard Elementary School. The teachers and principal of the school have been informed of this research and are supportive of our efforts. Through these interviews, we will learn more about the informal strategies that students develop to solve arithmetic problems. Information from this and other research will eventually be used to analyze how children develop concepts of numbers and operations and to design curriculum and teacher in-service programs that enable teachers to build on children's informal knowledge.

These interviews will consist of individual interviews with students in a quiet place away from their classrooms. Interviews will be videotaped for post-analysis and possibly for educational purposes. Both you and your child must give your consent before any interviews can be carried out. Students will be told by their teachers that although we would like their help, they are not required to participate and that the interview has nothing to do with the school or their grades, nor will teachers or administrators have access to the video tapes. Even if your child signs the consent form, she/he reserves the right to later withdraw from the study.

Please discuss this with your child and send the form below to school with your child. If you have any questions, please feel free to contact Sheryl Cabral at 794-8172.

Sincerely,

Dr. Vicky Kouba, Aileen O'Brien, and Sheryl Cabral
Dept. of Educational Theory and Practice
SUNY-Albany

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I _____ do not _____ do (check one) grant permission for my child _____ to be interviewed. (child's name)

(check one)

_____ I grant permission for the video tape of the interview with my child to be used for future educational purposes such as teacher training courses or workshops.

_____ I want the video tape of my child destroyed after it has been analyzed by the above named researchers.

_____ _____
(Date) (Signature)

I understand that I can withdraw from this study at any time, and that my performance in this interview will in no way affect my academic standing at school.

_____ _____
(Date) (Student's Signature)